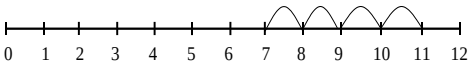
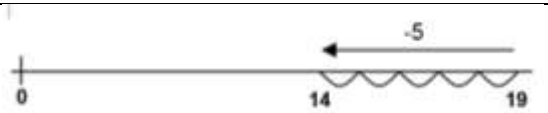
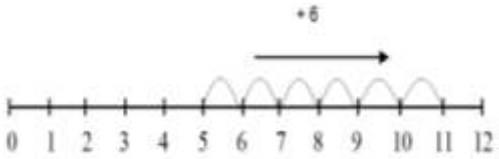
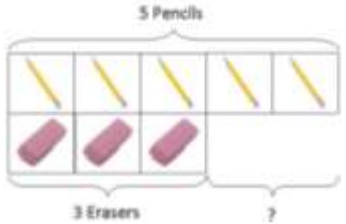
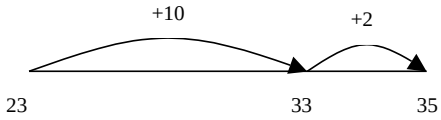
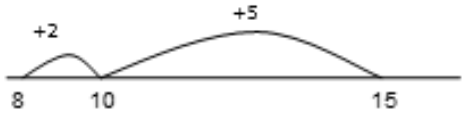
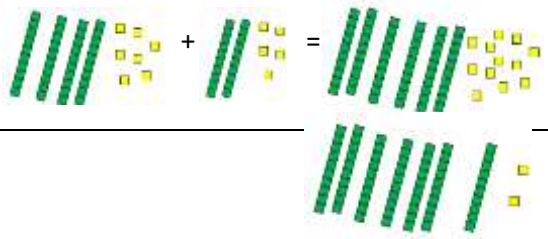
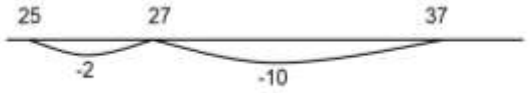
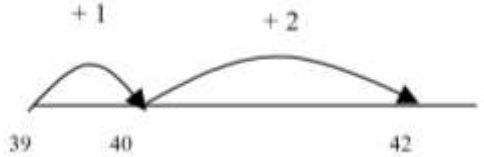
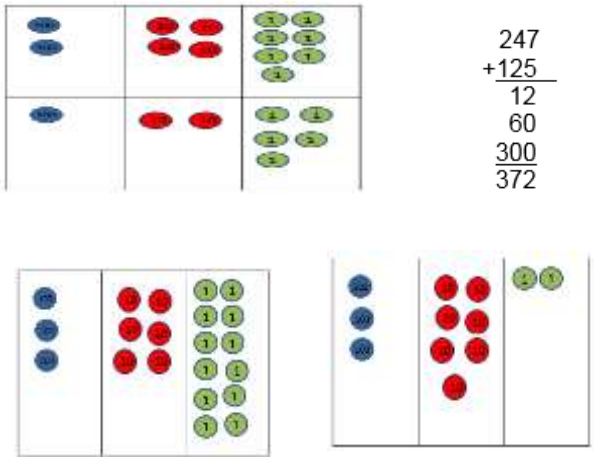
Addition and subtraction

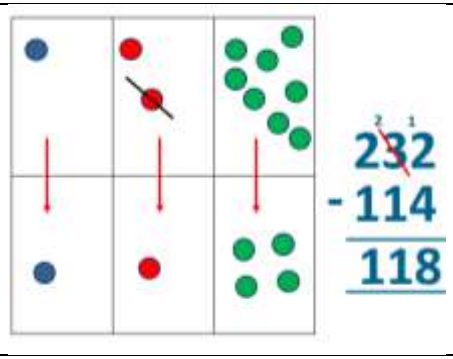
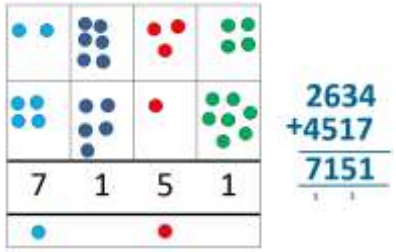
- Add and subtract numbers using concrete objects, pictorial representations.
- All number facts for all numbers 1-10 are secure by the end of Year 1 in order to progress and consolidate decimal numbers in KS2.
- From Year 2, inverse of addition and subtraction to be used to check both written and mental calculations.
- All methods must be supported using a range of counting resources such as numicon, cubes, dienes, coins, counters, bead strings, number tracks, number lines, number squares, cuisinaire, dice, place value counters, double sided counters
- Develop visualising an increasing amount of numbers

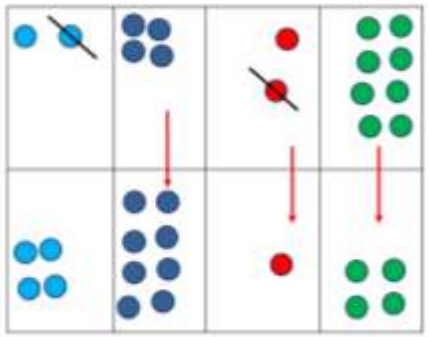
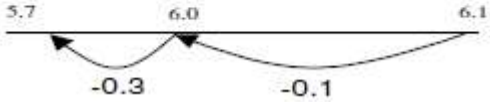
	Recall/mental	Mental strategies	Written	Examples	Problem solving
Year 1	To represent and use number bonds and related subtraction facts for all numbers 1-20. Add and subtract one-digit and two-digit numbers to 20, including zero.	– count on or back; – biggest number first; – begin to bridge; – use known number facts and place value to add or subtract pairs of single-digit numbers; – add 9 to single-digit numbers by adding 10 then subtracting 1; – identify near doubles, using doubles already known; – use patterns of similar calculations eg. 2 + 3 = 5 so 2 + 4 must be 6 (4 is one more than 3)	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs Add and subtract numbers including: *a one digit number and ones * a two-digit number and ones *Written methods are informal at this stage – see mental methods for expectation of calculations)	+Counting all leading to counting on.  +Counting on using a number track  +Counting on using a number line 7+4  –Taking away as counting back 	Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9; 20 - □ = 9; 15 – 9 = □; □ - □ = 11; 16 – 0 = □

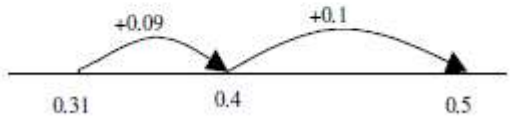
				 – Taking away as finding a difference  The bar model – concrete objects used which can be moved, then cards with pictures before pictoral representations. 	
Year 2	Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100. Recall and use number bonds for multiples of 5 totalling 60 (to support telling time to nearest 5 minutes)	-count on or back in tens or ones; – find a small difference by counting up from the smaller to the larger number; – biggest number first; – add three small numbers by putting the largest number first and/or find a pair totalling 10 or 20; – partition additions into tens and units then recombine;	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs Add and subtract numbers including: * a two-digit number and ones * a two-digit number and tens	+ Counting on in tens and ones  + Partitioning and bridging though 10  + Partitioning 47 25 60 + 12 	Solve problems with addition and subtraction: * using concrete objects and pictorial representations, including those involving numbers, quantities and measures * applying their increasing knowledge of mental and written methods

	Add and subtract numbers mentally, including: * a two-digit number and ones * a two-digit number and tens * two two-digit numbers * adding three one-digit numbers	– bridge through 10 or 20; – use known number facts and place value to add or subtract pairs of numbers; – partition into '5 and a bit' when adding 6, 7, 8 or 9, then recombine; – add or subtract 9, 19, 11 or 21 by rounding and compensating; – identify near doubles; – use patterns of similar calculations; eg. 2 + 3 = 5 so 2 + 4 must be 6 (4 is one more than 3) – use the relationship between addition and subtraction.	*Written methods are informal at this stage	Leading to exchanging **+Expanded written method** 40 + 7 + 20 + 5 = 40+20 + 7 + 5 = 60 + 12 = 72 40 + 7 + 20 + 5 60 + 12 = 72 – Counting back to take away  – Counting on to take away.  Link to finding a difference. Continue to use the bar model. Towards a written method for subtraction Use dienes to partition and take away, first without exchange, then with exchange.	Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change.
Year 3	Recall and use addition and subtraction facts for 100 (multiples of 5 and 10) Derive and use addition and	– count on or back in tens or ones; – find a small difference by counting up from the smaller to the larger number; – biggest number first;	Add and subtract numbers with up to three digits, using informal methods, progressing to formal written methods of	**+Partition into tens and ones** Partition both numbers and recombine. Count on by partitioning the second number only e.g. 247 + 125 = 247 + 100 + 20+ 5 = 347 + 20 + 5 = 367 + 5 = 372	Solve problems, including missing number problems (including some 2-step) using number facts, place value, and more complex

	subtraction facts for 100. Derive and use addition and subtraction facts for multiples of 100 totalling 1000. Add and subtract numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds Add and subtract fractions with the same denominator within 1 whole.	– add three or four small numbers by putting the largest number first and/or by finding pairs totalling 9, 10 or 11; – partition into tens and units then recombine; – bridge through a multiple of 10, then adjust; – use knowledge of number facts and place value to add or subtract pairs of numbers; – partition into '5 and a bit' when adding 6, 7, 8 or 9; – add or subtract mentally a 'near multiple of 10' to or from a three-digit number; – identify near doubles; – use patterns of similar calculations eg. 12 + 27 = 39 so 12 + 37 must be 49 (49 is ten more than 39); – say or write a subtraction statement corresponding to a given addition statement;	columnar addition and subtraction where appropriate.	**+Towards a Written Method for addition** Introduce columnar addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)  +Leading to formal columnar addition 247 + 125 372 1 **– Towards a written method for subtraction** Use dienes then place value counters to show column subtraction, without exchange, then with exchange using 3 digit numbers.	addition and subtraction

					
Year 4	Select a mental strategy appropriate for the calculation. Add and subtract mentally combinations of two and three digit numbers and decimals to one decimal place. Recall and use addition and subtraction facts for 100. Recall and use addition and subtraction facts for multiples of 100 totalling 1000.	Apply the same strategies as in year 3 but using decimals as well as whole numbers.	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.	+Columnar addition of 4 digit numbers modelled with place value counters.  – Progress to subtraction of 4 digit numbers with decomposition, using place value counters.	Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. Solve addition and subtraction problems involving missing numbers.

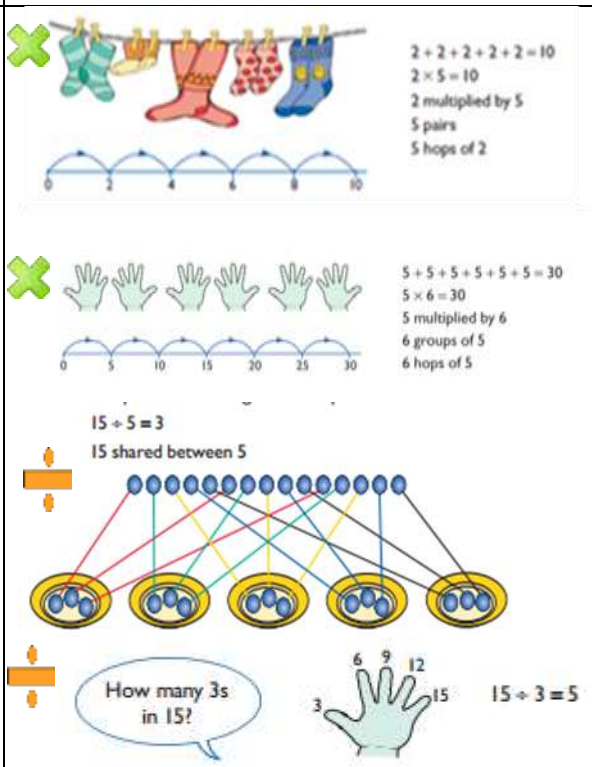
	Add and subtract fractions with the same denominator. **Derive** and use addition and subtraction facts for all numbers 1-10 (with decimal numbers to one decimal place)			 6232 - 4814 1418	
Year 5	Select a mental strategy appropriate for the numbers. Add and subtract numbers mentally with increasingly large numbers and decimals to two decimal places. 1 Add and subtract mentally with increasingly large numbers. **Derive** and use addition and subtraction facts for all numbers 1-10 (with decimal numbers to two decimal place) **Recall** and use addition and	Consolidate previous strategies but using two decimal places as well as whole numbers.	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) Move onto decimal numbers to 2 decimal places. Recognise mixed numbers and improper fractions and convert from one to the other.	135.62 + 54.69 190.31 1 1 1 Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers. **Use known number facts and place value to subtract** 6.1 – 0.4 = 5.7  	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. Solve addition and subtraction problems involving missing numbers.

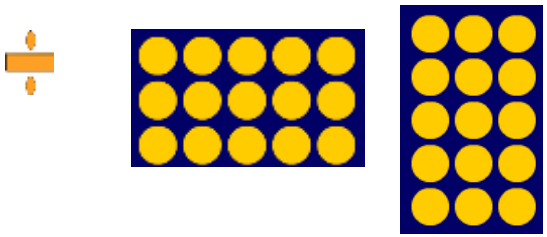
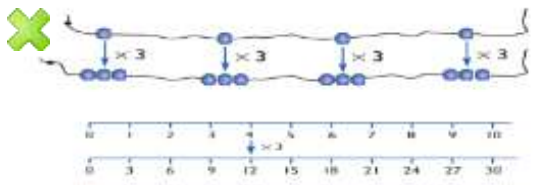
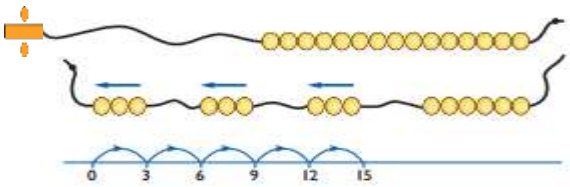
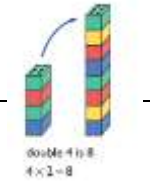
	subtraction facts for all numbers 1-10 decimal num to one decimal place)			Find a difference by counting up e.g. 0.5 – 0.31 = 0.19 This can be modelled on an empty number line (see complementary addition below).  	
Year 6	Select an appropriate mental strategy Add and subtract mentally combinations of two and three digit numbers and decimals to one decimal place. Perform mental calculations, including with mixed operations and large numbers. **Recall** and use + - and subtraction facts for all numbers 1-10 (with decimal numbers to two d. p.)	but using two decimal places as well as whole numbers.	Add and subtract whole numbers and decimals using formal written methods (columnar addition and subtraction) Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.	**<u>Written methods</u>** As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. Continue calculating with decimals, including those with different numbers of decimal places	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. Solve addition and subtraction problems involving missing numbers.

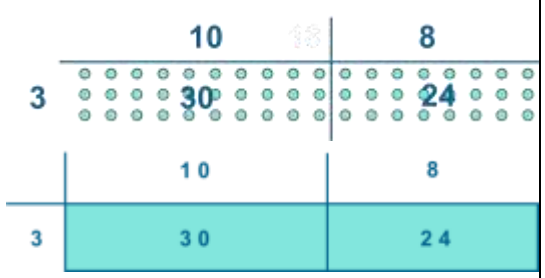
Calculations Policy for Multiplication and Division

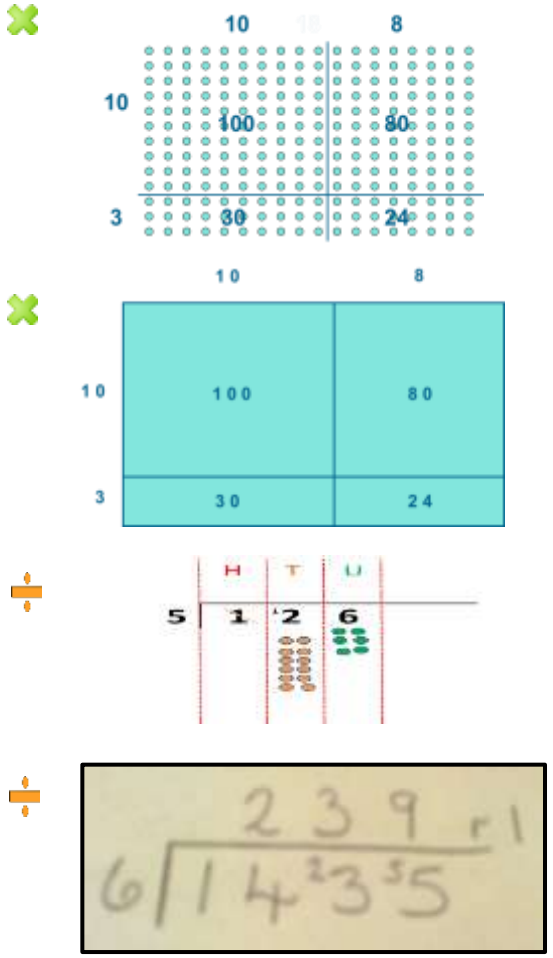
To aid in the learning of multiplication and division children should:

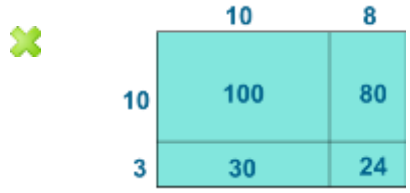
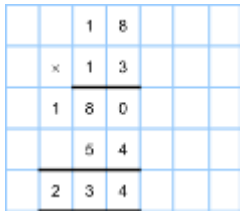
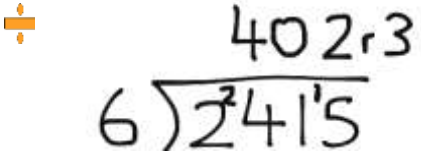
- Multiply and divide numbers using concrete objects and pictorial representations.
- Be secure in all multiplication facts, as designated by year group, for progression.
- From Year 2, use inverse of multiplication and division to check accuracy of calculations.
- be supported in all methods, using a range of counting resources such as numicon, cubes, dienes, coins, counters, bead strings, number tracks, number lines, number squares, cuiseneres, dice etc.

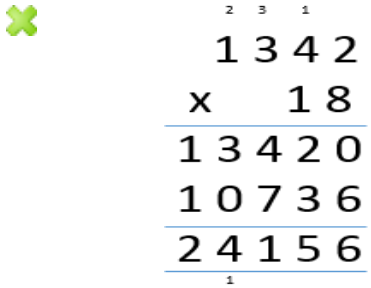
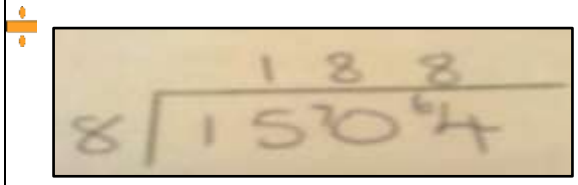
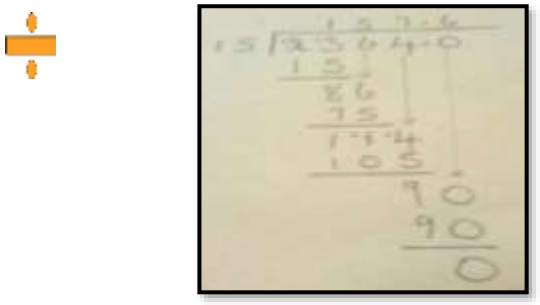
	Multiplication and Division Mental/Recall	**Strategies/Resources** (See Appendices for guidance)	**Written**	**Representations**	**Problem Solving**
Year 1	Recall and use doubles of all whole numbers to 10 and corresponding halves. Count in multiples of twos, fives and tens forwards and backwards. Group AND share small quantities- understanding the difference between the two concepts. Sharing Grouping Children should apply their counting skills to develop some understanding of grouping.	Doubling (x2) Halving (÷2)	**Written methods are informal at this stage – see mental methods for expectation of calculations*		Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

	Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3. 15 ÷ 5 = 3 There are 3 groups of 5.				
Year 2	Understand that multiplication is commutative but division is not. Understand that multiplication is the inverse of division and vice-versa. Count in steps of 2, 3 and 5 from 0 and when secure start from a different base number. Count in tens from any number, forwards and backwards. Recall and use x and division facts for the 2, 5 and 10 times tables, including recognising odd and even numbers. Derive and use doubles of simple two-digit numbers (numbers in which the ones total less than 10) Calculate mathematical statements for multiplication (using repeated addition) and division with the multiplication	Repeated Addition (e.g. 2+2+2+2) Doubling (x2) Halving (÷2) Use doubles and halves and halving as the inverse of doubling. Using known facts to find related facts	**Written methods are informal at this stage – see mental methods for expectation of calculations* Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.	Expressing multiplication as a number sentence using x Using understanding of the inverse and practical resources to solve missing number problems. 7 x 2 = □ □ = 2 x 7 7 x □ = 14 14 = □ x 7 □ x 2 = 14 14 = 2 x □ □ x ◯ = 14 14 = □ x ◯   	Solve problems involving multiplication and division *(including those with remainders),* using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

	tables and write them using the multiplication (x), division (÷) and equals signs (=).				
Year 3	Explain/reason that multiplication is commutative but division is not. Count from 0 in multiples of 4, 8, 50 and 100 and when secure start from a different base number. Recall and use multiplication and division facts for the 3, 4 and 8 times tables. Derive and use doubles of all numbers to 100 and corresponding halves. Derive and use doubles of all multiples of 50 to 500. Write and calculate mathematical statements for multiplication and division **using the multiplication tables that they know**, including for two-digit numbers multiplied by one-digit numbers, using mental methods.	Doubling (x2) Doubling again (x4) Doubling again (x8) Halving (÷2) Halving again (÷4) Halving again (÷8) Partitioning e.g. to multiply by 7, multiply by 5 and by 2 Moving Digits to multiply/divide by 10 Using known facts to find related facts e.g. 20 x 3 = 60, so 60 ÷ 3 = 20	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using informal methods and progressing to formal written methods. (appears also in Mental Methods)	Developing written methods using understanding of visual images Develop onto the grid method  **<u>÷ = signs and missing numbers</u>** Continue using a range of equations as in year 2 but with appropriate numbers. **<u>Grouping</u>** How many 6's are in 30? 30 ÷ 6 can be modelled as: **<u>Becoming more efficient using a number line</u>** Children need to be able to partition the dividend in different ways. 48 **÷ 4 = 12**  <u>Remainders</u> **49 ÷ 4 = 12 r1**  Sharing – 49 shared between 4. How many left over? Grouping – How many 4s make 49. How many are left over?	Solve problems, including missing number problems, involving multiplication and division *(and interpreting remainders)*, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

				Place value counters can be used to support children apply their knowledge of grouping.	
Year 4	Recall and use multiplication and division facts for multiplication tables up to 12 x 12. Count in multiples of 6, 7, 9, 25 and 1000 from 0 and when secure start from a different base number. Use knowledge of the 3 times table and doubling to derive and recall the 6 times table. Use partitioning to double and halve any number, including decimals to one decimal place. Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers. Recognise and use factor pairs and commutativity in mental calculations.	Doubling (x2) Doubling again (x4) Doubling again (x8) Halving (÷2) Halving again (÷4) Halving again (÷8) Doubling/halving can also be used for the relationships between 3, 6 and 12. Partitioning e.g. to multiply by 13, multiply by 10 and by 3 To multiply by 9, multiply by 10 and subtract '1 lot' of the multiple. Moving Digits to multiply/divide by 10 and 100. Using known facts to find related facts e.g. 200 x 3 = 600, so 600 ÷ 3 = 200	Multiply two-digit and three-digit numbers by a one-digit number using informal methods and progressing to formal written layout. Divide numbers up to 3 digits by a one-digit number using informal methods progressing to formal written method of short division and interpret remainders appropriately for the context.		Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, *division (including interpreting remainders),* integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Year 5	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Count forwards or backwards in steps of powers of 10 (10, 100, 1000 etc.) Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. Recall prime numbers up to 19 and derive prime numbers up to 100. Recall square numbers up to 144 and derive cube numbers. Apply understanding that a remainder is a fraction of a whole. Use partitioning to double or halve any number, including decimals to two decimal places.	Doubling (x2) Doubling again (x4) Doubling again (x8) Halving (÷2) Halving again (÷4) Halving again (÷8) Doubling/halving can also be used for the relationships between 3, 6 and 12. Partitioning e.g. to multiply by 24, multiply by 20 and by 4. To multiply by 99, multiply by 100 and subtract '1 lot' of the multiple. Moving Digits to multiply/divide by 10, 100 and 1000. Use multiplication/division facts to find equivalent decimal and decimal fraction remainders. e.g. 98 ÷ 4 = 24r2 = 24r2/4 = 24 ½ = 24.5 Using known facts to find related facts	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. Divide numbers up to 4 digits by a one-digit number using a formal written method of short division and interpret remainders appropriately for the context.	  Same method as Y4 	Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

Year 6	Associate a fraction with division and calculate decimal fraction equivalents. (e.g. 0.375 for a simple fraction – 3/8) Identify common factors, common multiples and prime numbers. Use partitioning to double or halve any number, including decimals to two places.	Doubling (x2) Doubling again (x4) Doubling again (x8) Halving (÷2) Halving again (÷4) Halving again (÷8) Doubling/halving can also be used for the relationships between 3, 6 and 12. Partitioning e.g. to multiply by 3.5, multiply by 3 and by 0.5. To multiply by 1.9, multiply by 2 and subtract '0.1 lot' of the multiple. Moving Digits to multiply/divide by 10, 100 and 1000. Use multiplication/division facts to find equivalent decimal and decimal fraction remainders. e.g. 24 3/8 = 0.375 (1/4 = 0.25, 1/8 = 0.125 so 3/8 = 0.125 x 3 = 0.375) Using known facts to find related facts	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using a formal written method of long multiplication. Divide numbers up to 4 digits by a two-digit number using a formal written method of short division where appropriate, interpreting remainders according to the context. Use written division methods in cases where the answer has up to two decimal places.	  Formal written methods – long and short division  	Solve problems involving addition, subtraction, multiplication and division